Name_____ Class_____ Date_____

Lesson 7-7 *(pp. 400–407)* Inverse Relations and Functions

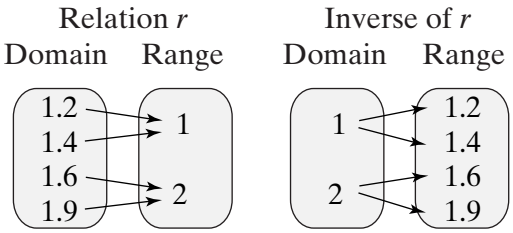
Lesson Objective	**NAEP 2005 Strand:** Algebra
1 Finding the inverse of a relation or a function	**Topic:** Patterns, Relations, and Functions
	Local Standards: _____

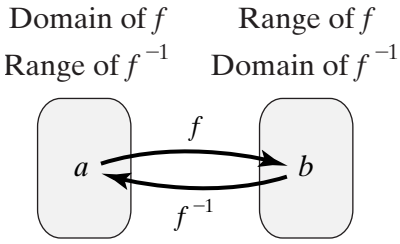
Vocabulary and Key Concepts

Composition of Inverse Functions

If f and f^{-1} are inverse functions, then $(f^{-1} \circ f)(x) =$ [x] and $(f \circ f^{-1})(x) =$ [x].

If a relation maps element a of its domain to element b of its range, the inverse relation **"undoes" the relation and maps *b* back to *a*.**





The inverse of function f is denoted by **f^{-1}.**

If a function f pairs a with b, then f^{-1} must **pair *b* with *a*.**

Examples

1 Finding the Inverse of a Relation

a. Find the inverse of relation m.

Relation m

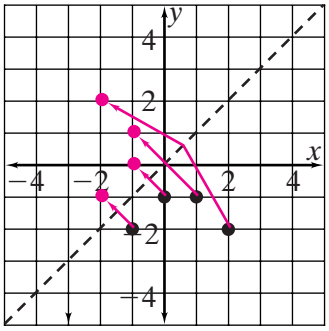
x	−1	0	1	2
y	−2	−1	−1	−2

Inverse of Relation m

x	**−2**	−1	**−1**	−2
y	−1	**0**	1	**2**

Interchange the x and y columns.

b. Graph m and its inverse on the same graph. Reverse the ordered pairs of relation m to graph the inverse of m.



Name_____ Class_____ Date _____

❷ Interchanging *x* and *y* Find the inverse of $y = x^2 - 2$.

$x = y^2 - 2$ **Interchange *x* and *y*.**

$x + \boxed{2} = y^2$ **Add 2 to each side.**

$\pm\sqrt{x + \boxed{2}} = \boxed{y}$ **Find the square root of each side.**

❸ Finding an Inverse Function Consider the function $f(x) = \sqrt{2x + 2}$.

a. Find the domain and range of f.

Since the radicand cannot be negative, the domain is the set of numbers greater than or equal to $\boxed{-1}$. Since the principal square root is nonnegative, the range is the set of $\boxed{\text{nonnegative}}$ numbers.

b. Find f^{-1}.

$\boxed{y} = \sqrt{2x + 2}$ **Rewrite the equation using *y*.**

$\boxed{x} = \sqrt{2\boxed{y} + 2}$ **Interchange *x* and *y*.**

$\boxed{x}^{\boxed{2}} = 2\boxed{y} + 2$ **Square both sides.**

$y = \dfrac{\boxed{x^2 - 2}}{\boxed{2}}$ **Solve for *y*.**

So, $f^{-1}(x) = \dfrac{\boxed{x^2 - 2}}{\boxed{2}}$.

c. Find the domain and range of f^{-1}.

The domain of f^{-1} equals the $\boxed{\text{range}}$ of f, which is the set of $\boxed{\text{nonnegative}}$ numbers.

Since $x^2 \geq 0$, $\frac{x^2 - 2}{2} \geq -1$. The range of f^{-1} is the set of numbers greater than or equal to $\boxed{-1}$.

The range of f^{-1} is the same as the $\boxed{\text{domain}}$ of f.

d. Is f^{-1} a function? Explain.

For each x in the domain of f^{-1}, there is only $\boxed{\text{one}}$ value of $f^{-1}(x)$.

So f^{-1} $\boxed{\text{is}}$ a function.

Check Understanding

1. a. Describe how the line $y = x$ is related to the graphs of m and its inverse in Example 1.

The graph of the inverse of *m* is a reflection in the line *y* = *x* of the graph of *m*.

b. Is relation m a function? Is the inverse of m a function?

yes; no

Name_____ Class_____ Date _____

Example

4 Composition of Inverse Functions For the function $f(x) = \frac{1}{2}x + 5$, find $(f^{-1} \circ f)(652)$ and $(f \circ f^{-1})(-\sqrt{86})$.

Since f is a linear function, so is f^{-1}. Therefore f^{-1} is a function. So $(f^{-1} \circ f)(652) =$ **652** and $(f \circ f^{-1})(-\sqrt{86}) =$ **$-\sqrt{86}$**.

Check Understanding

2. a. Does $y = x^2 + 3$ define a function? Is its inverse a function? Explain.

yes; no; For every *x*-value except 3 in the domain of the inverse there are two *y*-values.

b. Find the inverse of $y = 10 - 3x$. Is the inverse a function? Explain.

$y = \frac{1}{3}x + \frac{10}{3}$; It is a function because for each *x*-value there is only one *y*-value.

3. Let $f(x) = 10 - 3x$. Find each of the following.

a. the domain and range of f

domain: all real numbers; range: all real numbers

b. f^{-1}

$f^{-1}(x) = \frac{-x + 10}{3}$

c. the domain and range of f^{-1}

domain: all real numbers; range: all real numbers

d. $f^{-1}(f(3))$

3

4. For $f(x) = 5x + 11$, find $(f^{-1} \circ f)(777)$ and $(f \circ f^{-1})(-5802)$.

777; −5802